Electronic Companion to “Waste not want not: The environmental implications of quick response and upcycling”

In this document we provide supplementary analyses for the upcycling model with demand encroachment. We first solve for the firm's optimal decisions in stage 2 (in Section EC.1), and then analyze the firm's optimization problem in stage 1 (in Section EC.2). Section EC.3 presents the proofs of Proposition 9 and Lemma 3 in the paper.

EC.1. Firm's Optimization Problem in Stage 2

In Stage 2, given x and q and realized market size y, the focal firm chooses S and q_δ to maximize

$$\Pi_f^{U\dagger}(S, q_\delta, y) = p_f \cdot \min\{D_f^{U\dagger}(S, y), q + q_\delta\} - c_m x - cq - (c+\delta)q_\delta + wS,$$

subject to the constraints $S + q_\delta \leq x - q$, $S \geq 0$, $q_\delta \geq 0$.

For any given S, if $\alpha < 1 - p_f + p_s$, demand for the focal firm's product is (see §4.3 of the paper)

$$D_f^{U\dagger}(S,y) = \bar{D}_f^{U\dagger}(y) + D^\dagger(S,y) = \begin{cases} (1-p_f)y - \frac{S}{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})} \cdot (\frac{p_f-p_s}{1-\alpha} - p_f) & \text{if} \quad S < (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y \\ (1 - \frac{p_f-p_s}{1-\alpha})y, & \text{if} \quad S \geq (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y, \end{cases}$$

which simplifies to

$$D_f^{U\dagger}(S,y) = \begin{cases} (1-p_f)y - \alpha S & \text{if} \quad S < (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y \\ (1 - \frac{p_f-p_s}{1-\alpha})y, & \text{if} \quad S \geq (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y. \end{cases}$$

If $\alpha \geq 1 - p_f + p_s$, then demand for the focal firm's product becomes

$$D_f^{U\dagger}(S,y) = \bar{D}_f^{U\dagger}(y) + D^\dagger(S,y) = \begin{cases} (1-p_f)y - (\frac{1-p_f}{1-p_s/\alpha})S & \text{if} \quad S < (1 - \frac{p_s}{\alpha})y \\ 0, & \text{if} \quad S \geq (1 - \frac{p_s}{\alpha})y, \end{cases}$$

In either case, note that $D_f^{U\dagger}(S,y)$ weakly decreases in S and equals $(1-p_f)y$ at $S=0$. In the rest of this document we present the analysis for $\alpha < 1 - p_f + p_s$. The analysis for $\alpha \geq 1 - p_f + p_s$ is the same (with slightly different expressions) and hence omitted.

By Assumption 1 (i.e., $w \leq p_f - c - \delta$), the firm always prefers satisfying a unit of existing demand (via quick response) over selling a unit of fabric (or doing nothing). That is, its optimal production decision in stage 2 is given by $q_\delta = \min\{x - q, (D_f^{U\dagger}(S,y) - q)^+\}$. Substituting this into the firm's profit function yields

$$\Pi_f^{U\dagger}(S,y) = p_f \cdot \min\{D_f^{U\dagger}(S,y), x\} - c_m x - cq - (c+\delta)\min\{x - q, (D_f^{U\dagger}(S,y) - q)^+\} + wS, \qquad \text{(EC.1)}$$

which the firm maximizes by choosing S, subject to the constraint $0 \leq S \leq x - q - q_\delta = \min\{x - q, (x - D_f^{U\dagger}(S,y))^+\}$.

There are three cases, depending on how the realized market size y compares with x.

Case I) If $(1-p_f)y \geq x$: In this case, if the focal firm does not upcycle, it can use all of its fabric to fulfill demand since $D_f^{U\dagger}(0,y) = (1-p_f)y \geq x$. Thus, since we assume $p_f - c - \delta > w$, the firm chooses $S^{U\dagger}(x,q,y) = 0$. The focal firm's profit is $\Pi_f^{U\dagger}(S^{U\dagger}(x,q),y) = p_f x - (c+\delta)(x-q) - cq - c_m x$.

Case II) If $(1-p_f)y < x \le (1-\frac{p_s}{\alpha})y$: We have $D_f^{U\dagger}(S,y) < x$ for all S. In this case, the constraint $S \le x - D_f^{U\dagger}(S,y)$ is satisfied if and only if $S \le \bar{S}$, where $\bar{S} \doteq \frac{x-(1-p_f)y}{1-\alpha} < (\frac{p_f - p_s}{1-\alpha} - \frac{p_s}{\alpha})y$. Thus, the optimal S satisfies $0 \le S \le \min\{x-q, \frac{x-(1-p_f)y}{1-\alpha}\}$. There are three subcases.

II.a) If $\alpha x + (1-\alpha)q \le (1-p_f)y$, the above constraint further simplifies to $0 \le S \le \frac{x-(1-p_f)y}{1-\alpha}$, and it is easy to check that $q \le D_f^{U\dagger}(S,y) < x$ in this range. Thus, the profit function in Equation (EC.1) becomes

$$\begin{aligned}\Pi_f^{U\dagger}(S,y) &= p_f D_f^{U\dagger}(S,y) - c_m x - cq - (c+\delta)(D_f^{U\dagger}(S,y) - q) + wS \\ &= (p_f - c - \delta)((1-p_f)y - \alpha S - q) + (p_f - c)q - c_m x + wS \\ &= (p_f - c - \delta)((1-p_f)y - q) + (p_f - c)q - c_m x + (w - (p_f - c - \delta)\alpha)\, S.\end{aligned}$$

The optimal solution is $S^{U\dagger}(x,q,y) = 0$ if $w \le (p_f - c - \delta)\alpha$ and $S^{U\dagger}(x,q,y) = \frac{x-(1-p_f)y}{1-\alpha}$ otherwise. When $S^{U\dagger} = 0$, the focal firm uses some of its leftover fabric to satisfy its demand via quick response, and does not sell the rest (i.e., there is unused deadstock fabric). When $S^{U\dagger}(x,q,y) = \frac{x-(1-p_f)y}{1-\alpha}$, the firm uses some leftover fabric to satisfy its own demand via quick response, and sells the rest to the upcycling firm.

So

$$\Pi_f^{U\dagger}(S^{U\dagger}(x,q),y) = \begin{cases} (p_f - c - \delta)((1-p_f)y - q) + (p_f - c)q - c_m x, & \text{if} \quad w \le (p_f - c - \delta)\alpha, \\ (p_f - c - \delta)(x - q - S^{U\dagger}(x,q,y)) + (p_f - c)q - c_m x + wS^{U\dagger}(x,q), & \text{if} \quad w > (p_f - c - \delta)\alpha. \end{cases}$$

II.b) If $q \le (1-p_f)y < \alpha x + (1-\alpha)q$, the constraint becomes $0 \le S \le \min\{x-q, \frac{x-(1-p_f)y}{1-\alpha}\} = x - q$. The profit function in Equation (EC.1) becomes

$$\Pi_f^{U\dagger}(S,y) = \begin{cases} p_f D_f^{U\dagger}(S,y) - c_m x - cq - (c+\delta)(D_f^{U\dagger}(S,y) - q) + wS, & \text{if} \quad D_f^{U\dagger}(S,y) \ge q, \\ p_f D_f^{U\dagger}(S,y) - c_m x - cq + wS, & \text{if} \quad D_f^{U\dagger}(S,y) < q, \end{cases}$$

which reduces to

$$\Pi_f^{U\dagger}(S,y) = \begin{cases} (p_f - c - \delta)((1-p_f)y - q) + (p_f - c)q - c_m x + (w - (p_f - c - \delta)\alpha)\, S, & \text{if} \quad 0 \le S \le \frac{(1-p_f)y - q}{\alpha}, \\ p_f(1-p_f)y - c_m x - cq + (w - \alpha p_f)S, & \text{if} \quad \frac{(1-p_f)y-q}{\alpha} < S \le x - q. \end{cases}$$

The optimal solution is $S^{U\dagger}(x,q,y) = 0$ if $w \le (p_f - c - \delta)\alpha$, $S^{U\dagger}(x,q,y) = \frac{(1-p_f)y-q}{\alpha}$ if $(p_f - c - \delta)\alpha < w \le p_f\alpha$, and $S^{U\dagger}(x,q,y) = x - q$ if $w > p_f\alpha$. Note that in the second solution, $D_f^{U\dagger}(S^{U\dagger}(x,q,y),y) = q$ and $S^{U\dagger}(x,q,y) < x - q$, so the focal firm sells all of its finished goods, but there is leftover fabric that the firm neither uses nor sells. In the third solution, $D_f^{U\dagger}(S^{U\dagger}(x,q,y),y) < q$ and $S^{U\dagger}(x,q,y) = x - q$, so there is leftover finished goods but no leftover fabric.

II.c) If $(1-p_f)y < q$, the constraint is $0 \le S \le \min\{x-q, \frac{x-(1-p_f)y}{1-\alpha}\} = x - q$, and we have $D_f^{U\dagger}(S,y) < q$ for all S. Equation (EC.1) reduces to

$$\Pi_f^{U\dagger}(S,y) = p_f((1-p_f)y - \alpha S) - c_m x - cq + wS.$$

The optimal solution is $S^{U\dagger}(x,q,y) = 0$ if $w \le \alpha p_f$ and $S^{U\dagger}(x,q,y) = x - q$ if $w > \alpha p_f$. In both solutions, there is leftover finished goods. The first solution has leftover fabric $(x-q)$ while the second solution does not.

Case III) If $x > (1-\frac{p_s}{\alpha})y$: In this case, the constraint $S + D_f^{U\dagger}(S,y) \le x$ is satisfied if and only if $S \le x - (1-\frac{p_f-p_s}{1-\alpha})y$ (which is greater than $(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y$). Thus, the optimal solution satisfies $0 \le S \le \min\{x-q, x-(1-\frac{p_f-p_s}{1-\alpha})y\}$. There are again three subcases.

III.a) If $q \le (1-\frac{p_f-p_s}{1-\alpha})y$, the constraint becomes $S \le \min\{x-q, x-(1-\frac{p_f-p_s}{1-\alpha})y\} = x-(1-\frac{p_f-p_s}{1-\alpha})y$, and in this range, $q \le D_f^{U\dagger}(S,y) < x$. Equation (EC.1) reduces to

$$\Pi_f^{U\dagger}(S,y) = \begin{cases} (p_f-c-\delta)((1-p_f)y-q) + (p_f-c)q - c_m x + (w-(p_f-c-\delta)\alpha)\,S, & \text{if} \quad S \le (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y, \\ (p_f-c-\delta)((1-\frac{p_f-p_s}{1-\alpha})y-q) + (p_f-c)q - c_m x + wS, & \text{if} \quad (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y < S \le x-(1-\frac{p_f-p_s}{1-\alpha})y. \end{cases}$$

The optimal solution is $S^{U\dagger}(x,q,y) = x-(1-\frac{p_f-p_s}{1-\alpha})y$ if $w > (p_f-c-\delta)\alpha$. If $w \le (p_f-c-\delta)\alpha$, the focal firm compares $S=0$ with $S = x-(1-\frac{p_f-p_s}{1-\alpha})y$. We have $S^{U\dagger}(x,q,y) = 0$ if $w \le \frac{(p_f-c-\delta)(\alpha p_f - p_s)y}{(1-\alpha)x-(1-\alpha-p_f+p_s)y}$ and $S^{U\dagger}(x,q,y) = x-(1-\frac{p_f-p_s}{1-\alpha})y$ otherwise. When $S^{U\dagger}(x,q,y) = 0$ there is unused leftover fabric that the firm does not sell. When $S^{U\dagger}(x,q,y) = x-(1-\frac{p_f-p_s}{1-\alpha})y$ there are no leftover FGs or fabric.

III.b) If $(1-\frac{p_f-p_s}{1-\alpha})y < q \le (1-p_f)y$, the constraint becomes $0 \le S \le \min\{x-q, x-(1-\frac{p_f-p_s}{1-\alpha})y\} = x-q$. Equation (EC.1) reduces to

$$\Pi_f^{U\dagger}(S,y) = \begin{cases} (p_f-c-\delta)((1-p_f)y-q) + (p_f-c)q - c_m x + (w-(p_f-c-\delta)\alpha)\,S, & \text{if} \quad S \le \frac{(1-p_f)y-q}{\alpha}, \\ p_f(1-p_f)y - c_m x - cq + (w-\alpha p_f)S, & \text{if} \quad \frac{(1-p_f)y-q}{\alpha} < S < \min\{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y, x-q\}, \\ p_f(1-\frac{p_f-p_s}{1-\alpha})y - c_m x - cq + wS, & \text{if} \quad (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y \le S \le x-q. \end{cases}$$

There are two cases. (i) If $x-q \le (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y$, then the optimal solution is $S^{U\dagger}(x,q,y) = x-q$ if $w > p_f\alpha$, $S^{U\dagger}(x,q,y) = \frac{(1-p_f)y-q}{\alpha}$ if $(p_f-c-\delta)\alpha < w \le p_f\alpha$, and $S^{U\dagger}(x,q,y) = 0$ if $w \le (p_f-c-\delta)\alpha$. Solution 1 has leftover FGs ($q - D_f^{U\dagger} > 0$) but no leftover fabric; Solution 2 has no leftover FGs, no quick response, and some unused/unsold leftover fabric ($x-q-S^{U\dagger}(x,q,y) > 0$); Solution 3 has no leftover FGs, some leftover fabric used for quick response ($D_f^{U\dagger} - q > 0$), and some unused/unsold leftover fabric ($x-q-S^{U\dagger}(x,q,y) > 0$).

(ii) If $x-q > (\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y$, there are three sections. When $w > p_f\alpha$, the optimal solution is $S^{U\dagger}(x,q,y) = x-q$. When $(p_f-c-\delta)\alpha < w \le p_f\alpha$, the firm compares

$$\Pi_f^{U\dagger}(\frac{(1-p_f)y-q}{\alpha}, y) = p_f q - c_m x - cq + w\frac{(1-p_f)y-q}{\alpha}$$

with

$$\Pi_f^{U\dagger}(x-q,y) = p_f(1-\frac{p_f-p_s}{1-\alpha})y - c_m x - cq + w(x-q).$$

The difference is

$$\Pi_f^{U\dagger}(\frac{(1-p_f)y-q}{\alpha}, y) - \Pi_f^{U\dagger}(x-q,y) = p_f q - w(x-q+\frac{q}{\alpha}) + \frac{w}{\alpha}(1-p_f)y - p_f(1-\frac{p_f-p_s}{1-\alpha})y.$$

By analyzing the sign of the difference, we find that the focal firm chooses $S^{U\dagger}(x,q,y) = \frac{(1-p_f)y-q}{\alpha}$ if $y \ge \frac{\alpha(x-q)+q}{1-p_f}$ or $\{y < \frac{\alpha(x-q)+q}{1-p_f}, w \le \frac{\alpha p_f(q-(1-\frac{p_f-p_s}{1-\alpha})y)}{\alpha(x-q)+q-(1-p_f)y}\}$, and $S^{U\dagger}(x,q,y) = x-q$ otherwise.

When $w \le (p_f-c-\delta)\alpha$, the firm compares

$$\Pi_f^{U\dagger}(0,y) = p_f(1-p_f)y - c_m x - cq - (c+\delta)((1-p_f)y-q)$$

with

$$\Pi_f^{U\dagger}(x-q,y) = p_f(1-\frac{p_f-p_s}{1-\alpha})y - c_m x - cq + w(x-q).$$

The firm chooses $S^{U\dagger}(x,q,y) = 0$ if $\Big(p_f(\frac{p_f-p_s}{1-\alpha} - p_f) - (c+\delta)(1-p_f)\Big)y \ge w(x-q) - (c+\delta)q$, and $S^{U\dagger}(x,q,y) = x-q$ otherwise.

III.c) If $(1-p_f)y < q$, Equation (EC.1) reduces to

$$\Pi_f^{U\dagger}(S,y) = \begin{cases} p_f((1-p_f)y - \alpha S) - cq - c_m x + wS, & \text{if} \quad S \le \min\{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y, x-q\}, \\ p_f(1-\frac{p_f-p_s}{1-\alpha})y - cq - c_m x + wS, & \text{if} \quad \min\{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})y, x-q\} \le S \le x-q. \end{cases}$$

When $y \ge \frac{x-q}{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})}$, the optimal solution is $S^{U\dagger}(x,q,y) = 0$ if $w \le \alpha p_f$ and $S^{U\dagger}(x,q,y) = x-q$ if $w > \alpha p_f$.

When $y < \frac{x-q}{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})}$ and $w > \alpha p_f$, the optimal solution is $S^{U\dagger}(x,q,y) = x-q$.

When $y < \frac{x-q}{(\frac{p_f-p_s}{1-\alpha} - \frac{p_s}{\alpha})}$ and $w \le \alpha p_f$, the firm compares

$$\Pi_f^{U\dagger}(0,y) = p_f(1-p_f)y - cq - c_m x$$

with

$$\Pi_f^{U\dagger}(x-q,y) = p_f(1-\frac{p_f-p_s}{1-\alpha})y - cq - c_m x + w(x-q).$$

The optimal solution is $S^{U\dagger}(x,q,y) = 0$ if $w \le \frac{p_f(\alpha p_f - p_s)y}{(1-\alpha)(x-q)}$, and $S^{U\dagger}(x,q,y) = x-q$ otherwise.

Summary of Optimal Upcycling Decision in Stage 2

Summarizing the analyses above, we can conclude that for any given x, q and realized market size y, there exists a unique pair of thresholds $W_1(x,q,y)$ and $W_2(x,q,y)$ such that the optimal upcycling decision is given by:

$$S^{U\dagger}(x,q) = \begin{cases} 0 & \text{if } w \le W_1(x,q,y), \\ \max\{0, \frac{(1-p_f)y-q}{\alpha}\} & \text{if } W_1(x,q,y) < w < W_2(x,q,y), \\ \min\{x-q, x-(1-\frac{p_f-p_s}{1-\alpha})y, \frac{(x-(1-p_f)y)^+}{1-\alpha}\} & \text{if } w \ge W_2(x,q,y). \end{cases}$$

When $w \le W_1(x,q,y)$, the firm does not upcycle at all. When $W_1(x,q,y) < w < W_2(x,q,y)$, the firm upcycles only a portion of its on-hand fabric such that it retains enough demand to sell out its inventory of finished goods. When $w \ge W_2(x,q,y)$, the firm upcycles as much on-hand fabric as possible, so that every unit of on-hand fabric is either upcycled or used to satisfy demand via quick response.

The thresholds $W_1(x,q,y)$ and $W_2(x,q,y)$ are illustrated in Figure EC.1, and described in more detail below. Specifically, there are six cases, as illustrated in the six sub-figures of Figure EC.1.

We define the following expressions: $w_1(x,q,y) \doteq \frac{p_f(\alpha p_f - p_s)y}{(1-\alpha)(x-q)}$, $w_2(x,q,y) \doteq \frac{\alpha p_f\big(q-(1-\frac{p_f-p_s}{1-\alpha})y\big)}{\alpha x+(1-\alpha)q-(1-p_f)y}$, $w_3(x,q,y) \doteq \frac{p_f\cdot(q-(1-\frac{p_f-p_s}{1-\alpha})y)+(p_f-c-\delta)((1-p_f)y-q)}{x-q}$, $w_4(x,q,y) \doteq \frac{(p_f-c-\delta)(\alpha p_f-p_s)y}{(1-\alpha)x-(1-\alpha-p_f+p_s)y}$; $r_1 \doteq 1+\frac{\alpha p_f-p_s}{\alpha(1-\alpha)(1-p_f)}$, $r_2 \doteq 1+\frac{p_f(\alpha p_f-p_s)}{\alpha(1-\alpha)(1-p_f)(p_f-c-\delta)}$, and $r_3 \doteq 1+\frac{\alpha p_f-p_s}{\alpha(1-\alpha-p_f+p_s)}$.

Case (a): When $1 \le \frac{x}{q} \le r_1$, we have $\frac{\alpha(1-\alpha)(x-q)}{\alpha p_f-p_s} \le \frac{q}{1-p_f} \le \frac{\alpha x+(1-\alpha)q}{1-p_f} \le \frac{x}{1-p_f}$, and also $\frac{x}{1-p_s/\alpha} \le \frac{\alpha x+(1-\alpha)q}{1-p_f} \le \frac{q}{1-\frac{p_f-p_s}{1-\alpha}}$. Hence, the thresholds are: $W_1 = W_2 = w_1(x,q,y)$ for $y < \frac{\alpha(1-\alpha)(x-q)}{\alpha p_f-p_s}$, $W_1 = W_2 = \alpha p_f$ for $y \in [\frac{\alpha(1-\alpha)(x-q)}{\alpha p_f-p_s}, \frac{q}{1-p_f}]$, $W_1 = \alpha(p_f-c-\delta)$ and $W_2 = \alpha p_f$ for $y \in [\frac{q}{1-p_f}, \frac{\alpha x+(1-\alpha)q}{1-p_f}]$, $W_1 = W_2 = \alpha(p_f-c-\delta)$ for $y \in [\frac{\alpha x+(1-\alpha)q}{1-p_f}, \frac{x}{1-p_f}]$, and $W_1 = W_2 = 0$ for $y > \frac{x}{1-p_f}$.

Case (b): When $r_1 < \frac{x}{q} \le \min\{r_2, r_3\}$, we have $\frac{q}{1-p_f} \le \frac{\alpha(1-\alpha)(x-q)}{\alpha p_f - p_s} \le \frac{\alpha x+(1-\alpha)q}{1-p_f} \le \frac{x}{1-p_f}$, and also $\frac{x}{1-p_s/\alpha} \le \frac{\alpha x+(1-\alpha)q}{1-p_f} \le \frac{q}{1-\frac{p_f-p_s}{1-\alpha}}$. Hence, the thresholds are: $W_1 = W_2 = w_1(x,q,y)$ for $y \le \frac{q}{1-p_f}$, $W_1 = \alpha(p_f - c - \delta)$ and $W_2 = w_2(x,q,y)$ for $y \in (\frac{q}{1-p_f}, \frac{\alpha(1-\alpha)(x-q)}{\alpha p_f - p_s}]$, $W_1 = \alpha(p_f - c - \delta)$ and $W_2 = \alpha p_f$ for $y \in (\frac{\alpha(1-\alpha)(x-q)}{\alpha p_f - p_s}, \frac{\alpha x+(1-\alpha)q}{1-p_f}]$, $W_1 = W_2 = \alpha(p_f - c - \delta)$ for $y \in (\frac{\alpha x+(1-\alpha)q}{1-p_f}, \frac{x}{1-p_f}]$, and $W_1 = W_2 = 0$ for $y > \frac{x}{1-p_f}$.

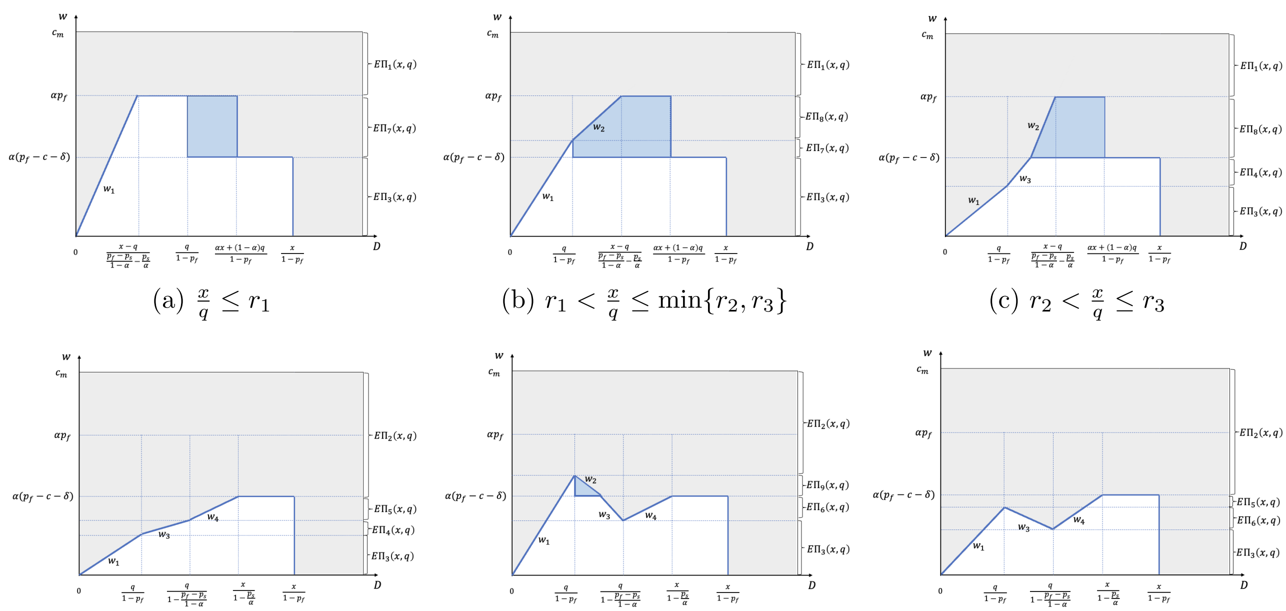
Case (c): When $r_2 < \frac{x}{q} \le r_3$, which is only possible for $c+\delta < \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)}$, the thresholds are the same as in Case (b) except that $W_1 = W_2 = w_3(x,q,y)$ for $y \in (\frac{q}{1-p_f}, \frac{(p_f-c-\delta)(\alpha x+(1-\alpha)q) - p_f(q-(1-\frac{p_f-p_s}{1-\alpha})y)}{(p_f-c-\delta)(1-p_f)}]$.

Case (d): when $\frac{x}{q} > r_3$ and $c+\delta < \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)}$ (i.e., $r_2 < r_3$), we have $\frac{q}{1-p_f} \le \frac{q}{1-\frac{p_f-p_s}{1-\alpha}} \le \frac{x}{1-p_s/\alpha} \le \frac{x}{1-p_f}$, and also $\frac{q}{1-\frac{p_f-p_s}{1-\alpha}} \le \frac{\alpha x+(1-\alpha)q}{1-p_f} \le \frac{x}{1-p_s/\alpha}$. The thresholds are: $W_1 = W_2 = w_1(x,q,y)$ for $y \le \frac{q}{1-p_f}$, $W_1 = W_2 = w_3(x,q,y)$ for $y \in (\frac{q}{1-p_f}, \frac{q}{1-\frac{p_f-p_s}{1-\alpha}}]$, $W_1 = W_2 = w_4(x,q,y)$ for $y \in (\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}, \frac{x}{1-p_s/\alpha}]$, $W_1 = W_2 = \alpha(p_f - c - \delta)$ for $y \in (\frac{x}{1-p_s/\alpha}, \frac{x}{1-p_f}]$, and $W_1 = W_2 = 0$ for $y > \frac{x}{1-p_f}$.

Case (e): when $r_3 < \frac{x}{q} \le r_2$ and $c+\delta \ge \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)}$ (i.e., $r_3 \le r_2$), the thresholds are the same as those in Case (d) except that $W_1 = \alpha(p_f - c - \delta)$ and $W_2 = w_2(x,q,y)$ for $y \in (\frac{q}{1-p_f}, \frac{(p_f-c-\delta)(\alpha x+(1-\alpha)q) - p_f(q-(1-\frac{p_f-p_s}{1-\alpha})y)}{(p_f-c-\delta)(1-p_f)}]$.

Case (f): when $\frac{x}{q} > r_2$ and $c+\delta \ge \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)}$ (i.e., $r_3 \le r_2$), the thresholds are the same as those in Case (d).

Table EC.1 summarizes the optimal upcycling decision $S^{U\dagger}(x,q,y)$, the corresponding profit function, and the deadstock outcomes under different values of x, q, y, and w.



(a) $\frac{x}{q} \le r_1$ (b) $r_1 < \frac{x}{q} \le \min\{r_2, r_3\}$ (c) $r_2 < \frac{x}{q} \le r_3$

(d) $\frac{x}{q} > r_3 \cap r_2 < r_3$ (e) $r_3 < \frac{x}{q} \le r_2 \cap r_2 \ge r_3$ (f) $\frac{x}{q} > r_2 \cap r_2 \ge r_3$

Figure EC.1: Optimal solution regions in stage 2. In each figure, $S^{U\dagger}(x,q,y) = 0$ in the white area, $S^{U\dagger}(x,q,y) = \max\{0, \frac{(1-p_f)y-q}{\alpha}\}$ in the blue area, and $S^{U\dagger}(x,q,y) = \min\{x-q, x-(1-\frac{p_f-p_s}{1-\alpha})y, \frac{(x-(1-p_f)y)^+}{1-\alpha}\}$ in the gray area.

Conditions		$S^{U\dagger}(x,q,y)$	Leftover FG	Leftover fabric	Profit function $\Pi_f^{U\dagger}(S,y)$
$w \geq W_2$	$y \in (\frac{x}{1-p_f}, +\infty)$	0	0	0	$p_f(1-p_f)y - cq - c_m x$
	$\frac{x}{q} \leq r_3$: $y \in (\frac{\alpha x+(1-\alpha)q}{1-p_f}, \frac{x}{1-p_f}]$	$\frac{x-(1-p_f)y}{1-p_f}$	0	0	$(p_f - c - \delta)(x - q - S) + (p_f - c)q - c_m x + wS$
	$\frac{x}{q} > r_3$: $y \in (\frac{x}{1-p_s/\alpha}, \frac{x}{1-p_f}]$				
	$\frac{x}{q} > r_3$: $y \in (\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}, \frac{x}{1-p_s/\alpha}]$	$x-(1-\frac{p_f-p_s}{1-\alpha})y$	0	0	$(p_f-c-\delta)\left((1-\frac{p_f-p_s}{1-\alpha})y-q\right)+(p_f-c)q-c_m x+wS$
	$\frac{x}{q} \leq r_3$: $y \in (\frac{\alpha(1-\alpha)(x-q)}{\alpha p_f - p_s}, \frac{\alpha x+(1-\alpha)q}{1-p_f}]$	$x-q$	>0	0	$p_f((1-p_f)y-\alpha S) - c_m x - cq + wS$
	$\frac{x}{q} \leq r_3$: $y \in (0, \frac{\alpha(1-\alpha)(x-q)}{\alpha p_f - p_s}]$	$x-q$	>0	0	$p_f(1-\frac{p_f-p_s}{1-\alpha})y - c_m x - cq + wS$
	$\frac{x}{q} > r_3$: $y \in (0, \frac{q}{1-\frac{p_f-p_s}{1-\alpha}}]$				
$W_1 < w < W_2$	$y \in (\frac{q}{1-p_f}, \frac{\alpha x+(1-\alpha)q}{1-p_f})$	$\frac{(1-p_f)y-q}{\alpha}$	0	>0	$p_f q - c_m x - cq + wS$
$w \leq W_1$	$y \in (\frac{q}{1-p_f}, \frac{x}{1-p_f}]$	0	0	>0	$(p_f - c - \delta)((1-p_f)y - q) + (p_f - c)q - c_m x$
	$y \in (0, \frac{q}{1-p_f}]$	0	>0	>0	$p_f(1-p_f)y - cq - c_m x$

Table EC.1: The firm's optimal upcycling decision, deadstock outcomes and profit function in stage 2.

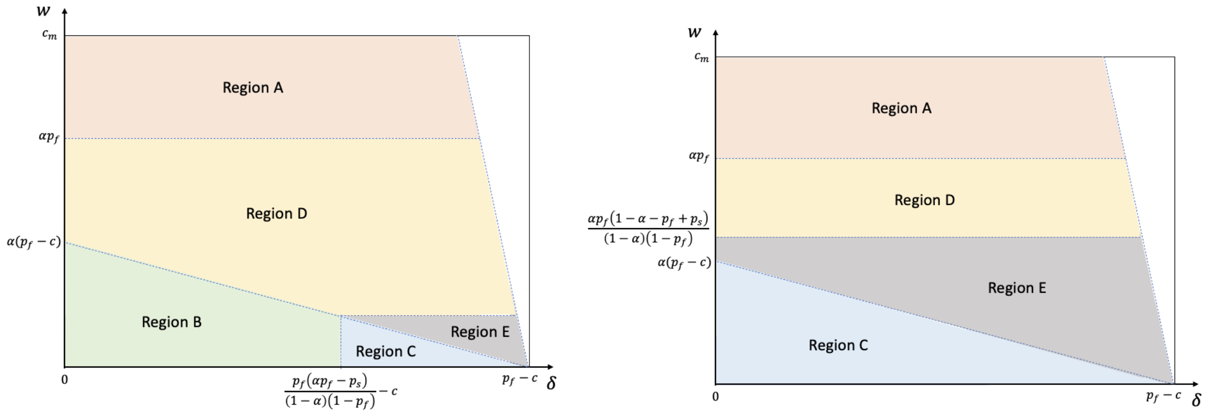
EC.2. Firm's Optimization Problem in Stage 1

The analysis in Section EC.1 provides the firm's optimal production and upcycling decisions in stage 2, for given x, q, and realized market size y. In stage 1, the firm chooses x and q to maximize its expected profit. As we can observe from Figure EC.1 and Table EC.1, the specific expression for the firm's expected profit function may vary depending on x and q, as well as the price/cost parameters (e.g., whether $w \geq \alpha p_f$). Overall, we distinguish five regions on the w-δ parameter space. In each region, the firm chooses (x, q) to maximize its expected profit $\mathbb{E}\Pi_f^{U\dagger}(x, q)$. The specific conditions for each region and the relevant expected profit functions are summarized in Table EC.2. Note that the condition $c+\delta < \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)}$ is obtained from $r_2 < r_3$. Figure EC.2 illustrates the regions. Note that the firm adopts full upcycling when $\mathbb{E}\Pi_f^{U\dagger}(x, q) = \mathbb{E}\Pi_1(x, q)$ or $\mathbb{E}\Pi_f^{U\dagger}(x, q) = \mathbb{E}\Pi_2(x, q)$ and selective upcycling in all other cases.

The specific expressions of the expected profit functions, i.e., $\mathbb{E}\Pi_i(x, q)$ for $i \in \{1, 2, ..., 9\}$, and the feasible solutions in each region are derived in the remaining part of this section.

Region	Expected profit function $\mathbb{E}\Pi_f^{U\dagger}(x, q)$
$w \geq \alpha p_f$ (Region A)	$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_1(x,q), & \text{if} \quad 1 \leq \frac{x}{q} \leq r_3 \\ \mathbb{E}\Pi_2(x,q), & \text{if} \quad \frac{x}{q} > r_3. \end{cases}$
$c+\delta < \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)} \cap w \leq \alpha(p_f - c - \delta)$ (Region B)	$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_3(x,q), & \text{if} \quad \frac{x-q}{q} \leq \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)} \\ \mathbb{E}\Pi_4(x,q), & \text{if} \quad \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)} < \frac{x-q}{q} < \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)} \\ \mathbb{E}\Pi_5(x,q), & \text{if} \quad \frac{x-q}{q} \geq \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)}. \end{cases}$
$c+\delta \geq \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)} \cap w \leq \alpha(p_f - c - \delta)$ (Region C)	$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_3(x,q), & \text{if} \quad 1 \leq \frac{x-q}{q} \leq \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)} \\ \mathbb{E}\Pi_6(x,q), & \text{if} \quad \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)} < \frac{x-q}{q} < \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} \\ \mathbb{E}\Pi_5(x,q), & \text{if} \quad \frac{x-q}{q} \geq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w}. \end{cases}$
$\max\{\alpha(p_f - c - \delta), \frac{\alpha p_f(1-\alpha-p_f+p_s)}{(1-\alpha)(1-p_f)}\} \leq w < \alpha p_f$ (Region D)	$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_2(x,q) & \text{if} \quad \frac{x}{q} \geq r_3 \\ \mathbb{E}\Pi_8(x,q), & \text{if} \quad \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1 < \frac{x}{q} < r_3 \\ \mathbb{E}\Pi_7(x,q), & \text{if} \quad 1 \leq \frac{x}{q} \leq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1. \end{cases}$
$\alpha(p_f - c - \delta) < w < \frac{\alpha p_f(1-\alpha-p_f+p_s)}{(1-\alpha)(1-p_f)})$ (Region E)	$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_2(x,q), & \text{if} \quad \frac{x}{q} \geq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1 \\ \mathbb{E}\Pi_9(x,q), & \text{if} \quad r_3 < \frac{x}{q} \leq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1 \\ \mathbb{E}\Pi_7(x,q), & \text{if} \quad 1 \leq \frac{x}{q} < r_3. \end{cases}$

Table EC.2: The firm's expected profit function in stage 1.



(a) $c \leq \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)}$ (b) $c > \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)}$

Figure EC.2: Solution regions in stage 1.

In each of the five possible regions on the (w, δ) parameter space, we derive the firm's expected profit function $\mathbb{E}\Pi_f^{U\dagger}(x,q)$ in stage 1 based on the results in Section EC.1. Then, in each region, we summarize the firm's optimization problem and present the feasible solutions and feasibility conditions. We assume $D \sim Uniform[0,1]$ in this analysis.

Region A: $w \geq \alpha p_f$

A.1) Summary of the optimization problem: We can show that the firm's expected profit function is given by

$$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_1(x,q), & \text{if} \quad 1 \leq \frac{x}{q} \leq r_3 \\ \mathbb{E}\Pi_2(x,q), & \text{if} \quad \frac{x}{q} > r_3. \end{cases}$$

The expressions for $\mathbb{E}\Pi_1(x,q)$ and $\mathbb{E}\Pi_2(x,q)$ are presented in A.2 and A.3, respectively. To find the (x,q) that maximizes $\mathbb{E}\Pi_f^{U\dagger}(x,q)$ (subject to the constraint $x \geq q$), we can compare the optimal solutions for two constrained maximization problems: (i) $\max_{x,q} \mathbb{E}\Pi_1(x,q)$ subject to the constraint $1 \leq \frac{x}{q} \leq r_3$, and (ii) $\max_{x,q} \mathbb{E}\Pi_2(x,q)$ subject to the constraint $\frac{x}{q} \geq r_3$. We note that the corner solution $\frac{x}{q} = r_3$ is never optimal (unless it coincides with an interior solution) because at $\frac{x}{q} = r_3$, $\mathbb{E}\Pi_1(x,q) = \mathbb{E}\Pi_2(x,q)$, $\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial q} = \frac{\partial \mathbb{E}\Pi_2(x,q)}{\partial q}$, and $\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial x} = \frac{\partial \mathbb{E}\Pi_2(x,q)}{\partial x}$. We denote the unique interior solution for $\max_{x,q} \mathbb{E}\Pi_i(x,q)$ by (x_i, q_i), $i \in \{1,2\}$. Similar notations are used for $i \in 3,4,...,9$ throughout this section.

Thus, the potentially feasible solutions to the firm's optimization problem in Region A are: (x_1, q_1), (x_2, q_2) and (x^B, q^B), where $x^B = q^B = \frac{(1-p_f)(p_f - c - c_m)}{p_f}$. To find the global optimum, it suffices to check the feasibility conditions of each of these solutions (provided in A.2 and A.3) and, if multiple solutions are feasible, compare their corresponding expected profits.

A.2) Suppose $1 \leq \frac{x}{q} \leq r_3$**:** Based on our analysis in Section EC.1, the firm's expected profit function in stage 1 is $\mathbb{E}\Pi_1(x,q) \doteq \int_0^{\frac{x-q}{\frac{p_f-p_s}{1-\alpha}-\frac{p_s}{\alpha}}} [p_f(1-\frac{p_f-p_s}{1-\alpha})y+w(x-q)]dF(y)+\int_{\frac{x-q}{\frac{p_f-p_s}{1-\alpha}-\frac{p_s}{\alpha}}}^{\frac{\alpha x+(1-\alpha)q}{1-p_f}} [p_f(1-p_f)y+(w-p_f\alpha)(x-q)]dF(y)+$

$\int_{\frac{\alpha x+(1-\alpha)q}{1-p_f}}^{\frac{x}{1-p_f}}[(p_f-c-\delta)(x-q)+p_f q-(p_f-c-\delta-w)\frac{x-(1-p_f)y}{1-\alpha}]dF(y)+\int_{\frac{x}{1-p_f}}^{+\infty}[p_f x-(c+\delta)(x-q)]dF(y)-cq-c_m x.$
Differentiating the function with respect to q and x and using $Y \sim Uniform[0,1]$, we obtain

$$\frac{\partial \mathbb{E}\Pi_1}{\partial q} = -\alpha p_f(\frac{x-q}{\frac{p_f-p_s}{1-\alpha}-\frac{p_s}{\alpha}}) + (\alpha p_f - w - c - \delta)(\frac{\alpha x + (1-\alpha)q}{1-p_f}) + \delta,$$

$$\frac{\partial \mathbb{E}\Pi_1}{\partial x} = \alpha p_f(\frac{x-q}{\frac{p_f-p_s}{1-\alpha}-\frac{p_s}{\alpha}}) + \frac{\alpha}{1-\alpha}(\alpha p_f - w - c - \delta)(\frac{\alpha x + (1-\alpha)q}{1-p_f}) + \frac{w+c+\delta-p_f}{1-\alpha}(\frac{x}{1-p_f}) + p_f - c - \delta - c_m.$$

Solving the first-order conditions (FOCs), $\frac{\partial \mathbb{E}\Pi_1}{\partial q} = 0$ and $\frac{\partial \mathbb{E}\Pi_1}{\partial x} = 0$, yields the interior solution (x_1, q_1), which is unique because the two derivatives are both linear in x and q.

We can also obtain the following expressions:

$$\begin{aligned}
\frac{\partial^2 \mathbb{E}\Pi_1}{\partial q^2} &= \frac{\alpha^2(1-\alpha)p_f}{\alpha p_f - p_s} + \frac{(1-\alpha)(\alpha p_f - w - c - \delta)}{1-p_f}, \\
\frac{\partial^2 \mathbb{E}\Pi_1}{\partial x^2} &= \frac{\alpha^2(1-\alpha)p_f}{\alpha p_f - p_s} + \frac{(1+\alpha)(w+c+\delta) - (1+\alpha+\alpha^2)p_f}{1-p_f}, \\
\frac{\partial^2 \mathbb{E}\Pi_1}{\partial x \partial q} &= -\frac{\alpha^2(1-\alpha)p_f}{\alpha p_f - p_s} + \frac{\alpha(\alpha p_f - w - c - \delta)}{1-p_f}, \\
\Delta_1 \doteq \frac{\partial^2 \mathbb{E}\Pi_1}{\partial q^2}\frac{\partial^2 \mathbb{E}\Pi_1}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_1}{\partial x \partial q})^2 &= \frac{-\alpha^2(1-\alpha)p_f^2}{(1-p_f)(\alpha p_f - p_s)} + \frac{(w+c+\delta-\alpha p_f)(p_f - w - c - \delta)}{(1-p_f)^2}, \\
x_1 - q_1 &= \frac{(1-p_f)(\alpha p_f - p_s)\left(\delta(c+c_m) - (p_f - c - c_m)(w + c - \alpha p_f)\right)}{(\alpha p_f - w - c - \delta)(p_f - c - w - \delta)(\alpha p_f - p_s) + (1-\alpha)\alpha^2 p_f^2 (1-p_f)}.
\end{aligned}$$

Overall, the interior solution (x_1, q_1) is feasible if and only if (iff) $x_1, q_1 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_1}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_1}{\partial x^2} < 0$, $\Delta_1 > 0$ and $1 \leq \frac{x_1}{q_1} \leq r_3$.

A.3) Suppose $\frac{x}{q} > r_3$: The firm's expected profit function in stage 1 is $\mathbb{E}\Pi_2(x,q) \doteq \int_0^{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}}[p_f(1-\frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}}^{\frac{x}{1-p_s/\alpha}}[p_f(1-\frac{p_f-p_s}{1-\alpha})y - (c+\delta)((1-\frac{p_f-p_s}{1-\alpha})y - q) + w(x-(1-\frac{p_f-p_s}{1-\alpha})y)]dF(y) + \int_{\frac{x}{1-p_s/\alpha}}^{\frac{x}{1-p_f}}[(p_f-c-\delta)(x-q)+p_f q-(p_f-c-\delta-w)\frac{x-(1-p_f)y}{1-\alpha}]dF(y)+\int_{\frac{x}{1-p_f}}^{+\infty}[p_f x-(c+\delta)(x-q)]dF(y)-cq-c_m x.$
The FOCs for the interior solution are:

$$\frac{\partial \mathbb{E}\Pi_2}{\partial q} = -(w+c+\delta)(\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}) + \delta = 0,$$

$$\frac{\partial \mathbb{E}\Pi_2}{\partial x} = \frac{\alpha(p_f - c - \delta - w)}{1-\alpha}(\frac{x}{1-p_s/\alpha}) + p_f - c - \delta - c_m - \frac{p_f - c - \delta - w}{1-\alpha}(\frac{x}{1-p_f}) = 0.$$

Solving the FOCs yields the unique interior solution:

$$q_2 = \frac{\delta}{w+c+\delta}(1 - \frac{p_f - p_s}{1-\alpha}), \qquad x_2 = \frac{(p_f - c - \delta - c_m)(1-\alpha)}{(p_f - c - \delta - w)(\frac{1}{1-p_f} - \frac{\alpha}{1-p_s/\alpha})}.$$

Note that $\frac{\partial^2 \mathbb{E}\Pi_2}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_2}{\partial x^2} = \frac{(p_f-c-\delta-w)}{1-\alpha}(\frac{\alpha}{1-p_s/\alpha} - \frac{1}{1-p_f}) < 0$ because $\alpha \in (\frac{p_s}{p_f}, 1)$, and $\frac{\partial^2 \mathbb{E}\Pi_2}{\partial q \partial x} = 0$. Therefore, solution (x_2, q_2) is feasible if and only if $\frac{x_2}{q_2} > r_3$.

Region B: $w < \alpha(p_f - c - \delta)$ **and** $c + \delta < \frac{p_f(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)}$ **(i.e.,** $r_2 < r_3$**)**

B.1) Summary of the optimization problem: We can show that the firm's expected profit is given by

$$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_3(x,q), & \text{if } \frac{x-q}{q} \leq \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)} \\ \mathbb{E}\Pi_4(x,q), & \text{if } \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)} < \frac{x-q}{q} < \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)} \\ \mathbb{E}\Pi_5(x,q), & \text{if } \frac{x-q}{q} \geq \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)}. \end{cases}$$

Similar to the case in Region A, we can rule out the corner solutions $\frac{x}{q} = 1 + \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)}$ and $\frac{x}{q} = 1 + \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)}$. We can also show that the interior solution to $\max_{x,q} \mathbb{E}\Pi_4(x,q)$ is never optimal. Thus, the potentially feasible solutions are (x_3, q_3), (x_5, q_5) and (x^B, q^B). We discuss the feasibility conditions in more detail below.

B.2) Suppose $\frac{x}{q} \leq 1 + \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)}$, which is obtained from $w \leq w_3(x, q, \frac{q}{1-p_f}) = \frac{p_f(\alpha p_f - p_s)q}{(x-q)(1-p_f)(1-\alpha)}$: The firm's expected profit function in stage 1 is $\mathbb{E}\Pi_3 \doteq \int_0^{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}} [p_f(1 - \frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}}^{\frac{q}{1-p_f}} p_f(1-p_f)ydF(y) + \int_{\frac{q}{1-p_f}}^{\frac{x}{1-p_f}} [p_f(1-p_f)y - (c+\delta)((1-p_f)y - q)]dF(y) + \int_{\frac{x}{1-p_f}}^{+\infty} [p_f x - (c+\delta)(x - q)]dF(y) - cq - c_m x$. The FOCs for the interior solution (x_3, q_3) are

$$\frac{\partial \mathbb{E}\Pi_3}{\partial q} = \delta - w\left(\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha} - p_f)}\right) - (c+\delta)(\frac{q}{1-p_f}) = 0,$$

$$\frac{\partial \mathbb{E}\Pi_3}{\partial x} = w\left(\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha} - p_f)}\right) + p_f - c - \delta - c_m - (p_f - c - \delta)(\frac{x}{1-p_f}) = 0.$$

We note that

$$\begin{aligned}
\frac{\partial^2 \mathbb{E}\Pi_3}{\partial q^2} &= \frac{w^2(1-\alpha)}{p_f(\alpha p_f - p_s)} - \frac{c+\delta}{1-p_f}, \\
\frac{\partial^2 \mathbb{E}\Pi_3}{\partial x^2} &= \frac{w^2(1-\alpha)}{p_f(\alpha p_f - p_s)} - \frac{p_f - c - \delta}{1-p_f}, \\
\frac{\partial^2 \mathbb{E}\Pi_3}{\partial x \partial q} &= -\frac{w^2(1-\alpha)}{p_f(\alpha p_f - p_s)}, \\
\Delta_3 \doteq \frac{\partial^2 \mathbb{E}\Pi_3}{\partial q^2}\frac{\partial^2 \mathbb{E}\Pi_3}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_3}{\partial x \partial q})^2 &= -\frac{w^2(1-\alpha)}{(\alpha p_f - p_s)(1-p_f)} + \frac{(p_f-c-\delta)(c+\delta)}{(1-p_f)^2}, \\
x_3 - q_3 &= \frac{(cp_f - (c+\delta)(c+c_m))(\alpha p_f - p_s)(1-p_f)}{(p_f-c-\delta)(c+\delta)(\alpha p_f - p_s) - (1-\alpha)(1-p_f)w^2}.
\end{aligned}$$

The interior solution (x_3, q_3) is feasible iff $x_3, q_3 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_3}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_3}{\partial x^2} < 0$, $\Delta_3 > 0$ and $1 \leq \frac{x_3}{q_3} \leq r_2$.

B.3) Suppose $1 + \frac{p_f(\alpha p_f - p_s)}{w(1-p_f)(1-\alpha)} < \frac{x}{q} \leq 1 + \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{w(1-\alpha-p_f+p_s)}$, where the upper bound is obtained from $w \leq w_3(x, q, \frac{q}{1-\frac{p_f-p_s}{1-\alpha}}) = \frac{(p_f-c-\delta)(\alpha p_f - p_s)q}{(1-\alpha-p_f+p_s)(x-q)}$: The firm's expected profit function is $\mathbb{E}\Pi_4 \doteq \int_0^{\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)}} [p_f(1 - \frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)}}^{\frac{x}{1-p_f}} [p_f(1-p_f)y - (c+\delta)((1-p_f)y - q)]dF(y) + \int_{\frac{x}{1-p_f}}^{+\infty} [p_f x - (c+\delta)(x-q)]dF(y) - cq - c_m x$. We have

$$\frac{\partial \mathbb{E}\Pi_4}{\partial q} = \alpha - (w+c+\delta)\left(\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha} - p_f) - (c+\delta)(1-p_f)}\right),$$

$$\frac{\partial \mathbb{E}\Pi_4}{\partial x} = w\left(\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)}\right)+p_f-c-\delta-c_m-(p_f-c-\delta)(\frac{x}{1-p_f}).$$

The interior solution is not feasible in this case because $\frac{\partial^2\mathbb{E}\Pi_4}{\partial q^2}>0$ always (note that $c+\delta<\frac{p_f(\alpha p_f-p_s)}{(1-\alpha)(1-p_f)}$ in Region B).

B.4) Suppose $\frac{x}{q}>1+\frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)}$: The firm's expected profit function is $\mathbb{E}\Pi_5 \doteq \int_0^{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}}[p_f(1-\frac{p_f-p_s}{1-\alpha})y+w(x-q)]dF(y)+\int_{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}}^{\frac{wx}{(p_f-c-\delta)(\frac{p_f-p_s}{1-\alpha}-p_f)+w(1-\frac{p_f-p_s}{1-\alpha})}}[(p_f-c-\delta-w)(1-\frac{p_f-p_s}{1-\alpha})y+(c+\delta)q+wx]dF(y)+\int_{\frac{wx}{(p_f-c-\delta)(\frac{p_f-p_s}{1-\alpha}-p_f)+w(1-\frac{p_f-p_s}{1-\alpha})}}^{\frac{x}{1-p_f}}[p_f(1-p_f)y-(c+\delta)((1-p_f)y-q)]dF(y)+\int_{\frac{x}{1-p_f}}^{+\infty}[p_fx-(c+\delta(x-q)]dF(y)-cq-c_mx$. The FOCs for the interior solution are

$$\frac{\partial \mathbb{E}\Pi_5}{\partial q}=\delta-(w+c+\delta)(\frac{q}{1-\frac{p_f-p_s}{1-\alpha}})=0,$$

$$\frac{\partial \mathbb{E}\Pi_5}{\partial x} = w\left(\frac{wx}{(p_f-c-\delta)(\frac{p_f-p_s}{1-\alpha}-p_f)+w(1-\frac{p_f-p_s}{1-\alpha})}\right)+p_f-c-\delta-c_m-(p_f-c-\delta)(\frac{x}{1-p_f})=0.$$

We can check that $\frac{\partial^2\mathbb{E}\Pi_5}{\partial q^2}<0$, $\frac{\partial^2\mathbb{E}\Pi_5}{\partial x^2}<0$ and $\frac{\partial^2\mathbb{E}\Pi_5}{\partial x\partial q}=0$. Thus, the interior solution (x_5,q_5) is feasible if and only if $x_5,q_5\geq 0$, and $\frac{x_5}{q_5}>1+\frac{(p_f-c-\delta)(\alpha p_f-p_s)}{(1-\alpha-p_f+p_s)w}$.

Region C: $w<\alpha(p_f-c-\delta)$ and $c+\delta\geq\frac{p_f(\alpha p_f-p_s)}{(1-\alpha)(1-p_f)}$ (i.e., $r_2\geq r_3$)

C.1) Summary of the optimization problem: We can show that the firm's expected profit is given by

$$\mathbb{E}\Pi_f^{U\dagger}(x,q)=\begin{cases}\mathbb{E}\Pi_3(x,q), & \text{if} \quad 1\leq\frac{x-q}{q}\leq\frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)}\\ \mathbb{E}\Pi_6(x,q), & \text{if} \quad \frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)}<\frac{x-q}{q}<\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}\\ \mathbb{E}\Pi_5(x,q), & \text{if} \quad \frac{x-q}{q}\geq\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}.\end{cases}$$

As in Region A, we can rule out the corner solutions $\frac{x}{q}=1+\frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)}$ and $\frac{x}{q}=1+\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}$. Thus, the potentially feasible solutions in Region C are (x_3,q_3), (x_6,q_6), (x_5,q_5) and (x^B,q^B). The feasibility conditions of (x_3,q_3) and (x_5,q_5) are similar to those discussed in Region B and hence omitted. In C.2 we discuss the feasibility conditions for (x_6,q_6).

C.2) Suppose $\frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)}<\frac{x-q}{q}<\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}$: Based on our analysis in EC.1, the firm's expected profit function is $\mathbb{E}\Pi_6 \doteq \int_0^{\frac{w(x-q)(1-\alpha)}{p_f(\alpha p_f-p_s)}}[p_f(1-\frac{p_f-p_s}{1-\alpha})y+w(x-q)]dF(y)+\int_{\frac{w(x-q)(1-\alpha)}{p_f(\alpha p_f-p_s)}}^{\frac{q}{1-p_f}}p_f(1-p_f)ydF(y)+\int_{\frac{q}{1-p_f}}^{\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)}}[p_f(1-p_f)y-(c+\delta)((1-p_f)y-q)]dF(y)+\int_{\frac{w(x-q)-(c+\delta)q}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)}}^{\frac{q(1-\alpha)}{1-\alpha-p_f+p_s}}[p_f(1-\frac{p_f-p_s}{1-\alpha})y+w(x-q)]dF(y)+\int_{\frac{q(1-\alpha)}{1-\alpha-p_f+p_s}}^{\frac{wx}{(p_f-c-\delta)(\frac{p_f-p_s}{1-\alpha}-p_f)+w(1-\frac{p_f-p_s}{1-\alpha})}}[(p_f-c-\delta-w)(1-\frac{p_f-p_s}{1-\alpha})y+(c+\delta)q+wx]dF(y)+\int_{\frac{wx}{(p_f-c-\delta)(\frac{p_f-p_s}{1-\alpha}-p_f)+w(1-\frac{p_f-p_s}{1-\alpha})}}^{\frac{x}{1-p_f}}[p_f(1-p_f)y-(c+\delta)((1-p_f)y-q)]dF(y)+\int_{\frac{x}{1-p_f}}^{+\infty}[p_fx-(c+\delta)(x-q)]dF(y)-cq-c_mx$.

The FOCs for the interior solution (x_6, q_6) are

$$\frac{\partial \mathbb{E}\Pi_6}{\partial q} = \delta - \frac{(c+\delta+w)q(1-\alpha)}{1-\alpha-p_f+p_s} + \frac{(w+c+\delta)\,(w(x-q)-(c+\delta)q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)-(c+\delta)(1-p_f)} - \frac{(c+\delta)q}{1-p_f} - \frac{w^2(x-q)(1-\alpha)}{p_f(\alpha p_f-p_s)} = 0$$

$$\frac{\partial \mathbb{E}\Pi_6}{\partial x} = \frac{w^2(1-\alpha)(x-q)}{p_f(\alpha p_f-p_s)} - \frac{(1-\alpha)(w^2(x-q)-(c+\delta)wq)}{p_f(\alpha p_f-p_s)-(c+\delta)(1-p_f)(1-\alpha)} + \frac{w^2(1-\alpha)x}{(p_f-c-\delta)(\alpha p_f-p_s)+w(1-\alpha-p_f+p_s)}$$
$$+ p_f - c - \delta - c_m - \frac{(p_f-c-\delta)x}{1-p_f} = 0.$$

The interior solution (x_6, q_6) is feasible iff $x_6, q_6 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_6}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_6}{\partial x^2} < 0$, $\Delta_6 \doteq \frac{\partial^2 \mathbb{E}\Pi_6}{\partial q^2} \cdot \frac{\partial^2 \mathbb{E}\Pi_6}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_6}{\partial q \partial x})^2 > 0$ and $1 + \frac{(p_f-c-\delta)(\alpha p_f-p_s)}{w(1-\alpha-p_f+p_s)} < \frac{x_6}{q_6} < 1 + \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}$.

Region D: $\max\{\alpha(p_f-c-\delta), \frac{\alpha p_f(1-\alpha-p_f+p_s)}{(1-\alpha)(1-p_f)}\} \leq w < \alpha p_f$

D.1) Summary of the optimization problem: We can show that the firm's expected profit is given by

$$\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_2(x,q) & \text{if } \ \frac{x}{q} \geq r_3 \\ \mathbb{E}\Pi_8(x,q), & \text{if } \ \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w} + 1 < \frac{x}{q} < r_3 \\ \mathbb{E}\Pi_7(x,q), & \text{if } \ 1 \leq \frac{x}{q} \leq \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w} + 1. \end{cases}$$

Similar to the previous regions, we can show that the potentially feasible solutions are (x_2, q_2), (x_8, q_8), (x_7, q_7), and (x^B, q^B).

D.2) Suppose $1 \leq \frac{x}{q} \leq \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}+1$: The firm's expected profit function is $\mathbb{E}\Pi_7 \doteq \int_0^{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}} [p_f(1-\frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}}^{\frac{q}{1-p_f}} p_f(1-p_f)ydF(y) + \int_{\frac{q}{1-p_f}}^{\frac{\alpha x+(1-\alpha)q}{1-p_f}} [p_f q + \frac{w}{\alpha}((1-p_f)y-q)]dF(y) + \int_{\frac{\alpha x+(1-\alpha)q}{1-p_f}}^{\frac{x}{1-p_f}} [p_f(\frac{(1-p_f)y-\alpha x}{1-\alpha}) - (c+\delta)(\frac{(1-p_f)y-\alpha x}{1-\alpha}-q) + w(\frac{x-(1-p_f)y}{1-\alpha})]dF(y) + \int_{\frac{x}{1-p_f}}^{+\infty} [p_f x - (c+\delta)(x-q)]dF(y) - cq - c_m x$. The FOCs for the interior solution are

$$\frac{\partial \mathbb{E}\Pi_7}{\partial q} = -w\left(\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}\right) + (p_f-c-\delta-\frac{w}{\alpha})(\frac{\alpha x+(1-\alpha)q}{1-p_f}) - (p_f-\frac{w}{\alpha})(\frac{q}{1-p_f}) + \delta = 0,$$

$$\frac{\partial \mathbb{E}\Pi_7}{\partial x} = w\left(\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}\right) - (p_f-c-\delta-\frac{w}{\alpha})(\frac{\alpha(x-q)}{1-p_f}) + (p_f-c-\delta)(1-\frac{x}{1-p_f}) - c_m = 0.$$

Note that

$$
\begin{aligned}
\frac{\partial^2 \mathbb{E}\Pi_7}{\partial q^2} &= \frac{w^2}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)} - \frac{\alpha p_f - w + (c+\delta)(1-\alpha)}{1-p_f},\\
\frac{\partial^2 \mathbb{E}\Pi_7}{\partial x^2} &= \frac{w^2}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)} - \frac{(1+\alpha)(p_f-c-\delta)-w}{1-p_f}\\
\frac{\partial^2 \mathbb{E}\Pi_7}{\partial q \partial x} &= -\frac{w^2}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)} + \frac{\alpha(p_f-c-\delta)-w}{1-p_f},\\
\Delta_7 = \frac{\partial^2 \mathbb{E}\Pi_7}{\partial q^2}\frac{\partial^2 \mathbb{E}\Pi_7}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_7}{\partial q \partial x})^2 &= -\frac{w p_f}{(1-p_f)^2} - \frac{(1-\alpha)w^2}{(1-p_f)(\alpha p_f - p_s)} + \frac{(p_f-c-\delta)(c+\alpha p_f+\delta)}{(1-p_f)^2},\\
x_7 - q_7 &= \frac{(1-p_f)(\alpha p_f - p_s)[\delta(c+c_m) - c(p_f-c-c_m)]}{(1-\alpha)(1-p_f)w^2 + w p_f(\alpha p_f - p_s) - (\alpha p_f - p_s)(p_f-c-\delta)(\alpha p_f + c + \delta)}.
\end{aligned}
$$

The solution (x_7, q_7) is feasible if and only if $x_7, q_7 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_7}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_7}{\partial x^2} < 0$, $\Delta_7 > 0$ and $1 \leq \frac{x_7}{q_7} \leq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1$.

D.3) Suppose $\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w}+1 < \frac{x}{q} < r_3$: The expected profit function is $\mathbb{E}\Pi_8 \doteq \int_0^{\frac{w(\alpha x+(1-\alpha)q)-\alpha p_f q}{(1-p_f)w-\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})}} [p_f(1-\frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{w(\alpha x+(1-\alpha)q)-\alpha p_f q}{(1-p_f)w-\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})}}^{\frac{\alpha x+(1-\alpha)q}{1-p_f}} [p_f q + \frac{w}{\alpha}((1-p_f)y-q)]dF(y) + \int_{\frac{\alpha x+(1-\alpha)q}{1-p_f}}^{\frac{x}{1-p_f}} [p_f(\frac{(1-p_f)y-\alpha x}{1-\alpha}) - (c+\delta)(\frac{(1-p_f)y-\alpha x}{1-\alpha} - q) + w(\frac{x-(1-p_f)y}{1-\alpha})]dF(y) + \int_{\frac{x}{1-p_f}}^{+\infty} [p_f x - (c+\delta)(x-q)]dF(y) - cq - c_m x$.

The FOCs for the unique interior solution are

$$
\frac{\partial \mathbb{E}\Pi_8}{\partial q} = -(w + p_f - \frac{w}{\alpha})\left(\frac{w(\alpha x+(1-\alpha)q)-\alpha p_f q}{(1-p_f)w-\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})}\right) + (p_f - c - \delta - \frac{w}{\alpha})(\frac{\alpha x+(1-\alpha)q}{1-p_f}) + \delta = 0,
$$

$$
\begin{aligned}
\frac{\partial \mathbb{E}\Pi_8}{\partial x} =& w\left(\frac{w(\alpha x+(1-\alpha)q)-\alpha p_f q}{(1-p_f)w-\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})}\right) + \frac{\alpha(p_f-c-\delta-w/\alpha)}{1-\alpha}(\frac{\alpha x+(1-\alpha)q}{1-p_f})\\
&+ p_f - c - \delta - c_m - \frac{(p_f-c-\delta-w)}{1-\alpha}(\frac{x}{1-p_f}) = 0.
\end{aligned}
$$

The interior solution (x_8, q_8) is feasible iff $x_8, q_8 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_8}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_8}{\partial x^2} < 0$, $\Delta_8 \doteq \frac{\partial^2 \mathbb{E}\Pi_8}{\partial q^2} \cdot \frac{\partial^2 \mathbb{E}\Pi_8}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_8}{\partial q \partial x})^2 > 0$ and $\frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w} + 1 < \frac{x_8}{q_8} < r_3$

Region E: $\alpha(p_f - c - \delta) \leq w < \frac{\alpha p_f(1-\alpha-p_f+p_s)}{(1-\alpha)(1-p_f)}$

E.1) Summary of the optimization problem: We can show that the firm's expected profit is given by

$$
\mathbb{E}\Pi_f^{U\dagger}(x,q) = \begin{cases} \mathbb{E}\Pi_2(x,q), & \text{if } \frac{x}{q} \geq \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w} + 1 \\ \mathbb{E}\Pi_9(x,q), & \text{if } r_3 < \frac{x}{q} < \frac{p_f(\alpha p_f-p_s)}{(1-p_f)(1-\alpha)w} + 1 \\ \mathbb{E}\Pi_7(x,q), & \text{if } 1 \leq \frac{x}{q} \leq r_3. \end{cases}
$$

The potentially feasible solutions are (x_2, q_2), (x_9, q_9) and (x_7, q_7). We discuss the feasibility conditions for (x_9, q_9) in E.2 below.

E.4) Suppose $r_3 < \frac{x}{q} \leq \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1$**:** The firm's expected profit function is given by $\mathbb{E}\Pi_9 \doteq \int_0^{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}} [p_f(1-\frac{p_f-p_s}{1-\alpha})y+w(x-q)]dF(y)+\int_{\frac{w(x-q)}{p_f(\frac{p_f-p_s}{1-\alpha}-p_f)}}^{\frac{q}{1-p_f}} p_f(1-p_f)ydF(y)+\int_{\frac{q}{1-p_f}}^{\frac{\alpha p_f q-w(\alpha x+(1-\alpha)q)}{\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})y-(1-p_f)w}} [p_f q+\frac{w}{\alpha}((1-p_f)y-q)]dF(y) + \int_{\frac{\alpha p_f q-w(\alpha x+(1-\alpha)q)}{\alpha p_f(1-\frac{p_f-p_s}{1-\alpha})y-(1-p_f)w}}^{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}} [p_f(1-\frac{p_f-p_s}{1-\alpha})y + w(x-q)]dF(y) + \int_{\frac{q}{1-\frac{p_f-p_s}{1-\alpha}}}^{\frac{x}{1-\frac{p_s}{\alpha}}} [(p_f-c-\delta-w)(1-\frac{p_f-p_s}{1-\alpha})y+(c+\delta)q+wx]dF(y)+\int_{\frac{x}{1-\frac{p_s}{\alpha}}}^{\frac{x}{1-p_f}} [p_f(\frac{(1-p_f)y-\alpha x}{1-\alpha})-(c+\delta)(\frac{(1-p_f)y-\alpha x}{1-\alpha}-q)+w(\frac{x-(1-p_f)y}{1-\alpha})]dF(y)+\int_{\frac{x}{1-p_f}}^{+\infty} [p_f x - (c+\delta(x-q)]dF(y) - cq - c_m x.$

The FOCs for the unique interior solution are

$$
\begin{aligned}
\frac{\partial \mathbb{E}\Pi_9}{\partial q} = \delta - \frac{(c+\delta)q}{1-p_f} - \frac{(p_f(\alpha p_f - p_s)(\alpha p_f - w) - (1-\alpha)(1-p_f)w^2)q}{(1-p_f)((1-p_f)(1-\alpha)w - \alpha p_f(1-\alpha-p_f+p_s))} + \frac{(1-\alpha)w(\alpha p_f - w + \alpha w)(x-q)}{(1-p_f)(1-\alpha)w - \alpha p_f(1-\alpha-p_f+p_s)} \\
- \frac{(1-\alpha)^2 w^2((1-p_f)q}{(1-p_f-\alpha+p_s)((1-p_f)(1-\alpha)w - \alpha p_f(1-\alpha-p_f+p_s))} - \frac{(1-\alpha)w^2(x-q)}{p_f(\alpha p_f - p_s)} - \frac{q(\alpha p_f - p_s)(c+\delta)}{(1-p_f)(1-p_f-\alpha+p_s)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathbb{E}\Pi_9}{\partial x} =& p_f - c - \delta - cm - \frac{(p_f-c-\delta)x}{1-p_f} + \frac{w(\alpha p_f - w + \alpha w)(1-\alpha)(x-q)}{\alpha p_f(1-\alpha-p_f+p_s) - (1-\alpha)(1-p_f)w)} \\
&+ \frac{(1-u)w^2(x-q)}{p_f(\alpha p_f - p_s)} - \frac{(p_s - \alpha + \alpha^2(1-p_f))(p_f - w - c - \delta)x + 2(p_s - \alpha p_f)(\alpha p_f - w)x}{(1-p_f)(\alpha - p_s)(1-\alpha)} \\
&+ \frac{(\alpha p_f - p_s)(1+\alpha)(c+\delta)}{(1-p_f)(1-\alpha)(\alpha - ps)} x - \frac{p_f x}{1-p_f} + \frac{\alpha(c+\delta+w)x}{\alpha - p_s} + \frac{(p_s - \alpha p_f)w(\alpha p_f - w + \alpha w)x}{(\alpha - p_s)(\alpha p_f(1-\alpha-p_f+p_s) - (1-\alpha)(1-p_f)w)}
\end{aligned}
$$

The interior solution (x_9, q_9) is feasible iff $x_9, q_9 \geq 0$, $\frac{\partial^2 \mathbb{E}\Pi_9}{\partial q^2} < 0$, $\frac{\partial^2 \mathbb{E}\Pi_9}{\partial x^2} < 0$, $\Delta_9 \doteq \frac{\partial^2 \mathbb{E}\Pi_9}{\partial q^2} \cdot \frac{\partial^2 \mathbb{E}\Pi_9}{\partial x^2} - (\frac{\partial^2 \mathbb{E}\Pi_9}{\partial q \partial x})^2 > 0$ and $r_3 < \frac{x_9}{q_9} < \frac{p_f(\alpha p_f - p_s)}{(1-p_f)(1-\alpha)w} + 1$

EC.3. Proofs for Lemma 3 and Proposition 9

Proof of Lemma 3: We first prove (ii). Based on the analyses in sections EC.1 and EC.2, we can see that when $w \geq \alpha p_f$ (i.e., Region A), the focal firm upcycles all deadstock fabric under any realized market size y. When $w \leq \alpha(p_f - c - \delta)$ (i.e., Regions B and C), the firm does not upcycle all of its deadstock fabric under some values of y, so that there exists deadstock fabric in expectation.

When $\alpha(p_f - c - \delta) < w < \alpha p_f$ (i.e., Regions D and E), the firm upcycles all deadstock fabric under any realized market size y if and only if (iff) (x_2, q_2) is the optimal solution. Note that this solution is feasible iff $w \geq \frac{\delta p_f(1-p_s/\alpha-\alpha+\alpha p_f)}{\delta(p_f-p_s/\alpha)+(p_f-c-c_m)(1-p_f)(1-\alpha)} - c - \delta$. Also note that when (x_2, q_2) is feasible, (x_8, q_8) and (x_9, q_9) are not. Hence, it suffices to consider only the case when (x_2, q_2) and (x_7, q_7) are feasible solutions, and show that the former dominates when w is sufficiently large. To do so, we prove that $\frac{d\mathbb{E}\Pi_7(x_7,q_7)}{dw} < \frac{d\mathbb{E}\Pi_2(x_2,q_2)}{dw}$, or equivalently (by inspecting the profit functions in EC.2), $\mathbb{E}S^{U\dagger}(x_7, q_7) < \mathbb{E}S^{U\dagger}(x_2, q_2)$. Now, for given x and q, let $S^0(x,q) \doteq \min\{x - q, x - (1 - \frac{p_f - p_s}{1-\alpha})y, \frac{(x-(1-p_f)y)^+}{1-\alpha}\}$ denote the amount upcycled if the firm upcycles all leftover deadstock fabric. Clearly, $\mathbb{E}S^{U\dagger}(x_7, q_7) < \mathbb{E}S^0(x_7, q_7)$. Moreover, since $\mathbb{E}S^0(x, q)$ increases in x and decreases in q, we have $\mathbb{E}S^0(x_7, q_7) < \mathbb{E}S^0(x_2, q_2)$ if $x_7 \leq x_2$ and $q_7 \geq q_2$.

To prove $x_7 \leq x_2$, we use $\hat{q}_7(x)$ to denote the solution to $\frac{\partial \mathbb{E}\Pi_7(x,q)}{\partial x}$ for given x. For the solution $(x, \hat{q}_7(x))$ to be feasible it needs to satisfy $\hat{q}_7(x) \geq x/r_3$. Substituting this condition into $\frac{\partial \mathbb{E}\Pi_7(x,q)}{\partial x}$ yields

$$\frac{\partial \mathbb{E}\Pi_7(x, \hat{q}_7(x))}{\partial x} \leq \left(\frac{w^2}{p_f(\alpha - p_s)} + \frac{(w - \alpha(p_f - c - \delta))(\alpha p_f - p_s)}{(1-\alpha)(1-p_f)(\alpha - p_s)}\right) x + p_f - c - \delta - c_m - (p_f - c - \delta)\frac{x}{1-p_f}.$$

Since

$$\frac{\partial \mathbb{E}\Pi_2(x,q)}{\partial x} = \left(\frac{\alpha(p_f - c - \delta - w)}{1-\alpha}\left(\frac{x}{1-p_s/\alpha}\right) + \frac{(w - \alpha(p_f - c - \delta))}{(1-\alpha)(1-p_f)}\right) x + p_f - c - \delta - c_m - (p_f - c - \delta)\frac{x}{1-p_f},$$

the difference between the two equations above is

$$\frac{\partial \mathbb{E}\Pi_7(x, \hat{q}_7(x))}{\partial x} - \frac{\partial \mathbb{E}\Pi_2(x,q)}{\partial x} = \frac{w}{\alpha - p_s}\left(\frac{w}{p_f} - \alpha\right) < 0.$$

Thus, substituting x_2, which satisfies $\frac{\partial \mathbb{E}\Pi_2(x,q)}{\partial x} = 0$ into $\frac{\partial \mathbb{E}\Pi_7(x,\hat{q}_7(x))}{\partial x}$ would result in $\frac{\partial \mathbb{E}\Pi_7(x_2,\hat{q}_7(x_2))}{\partial x} < 0$. This implies $x_7 < x_2$. We can show $q_7 > q_2$ similarly. This concludes the proof.

For (i), The firm's expected profit function under the upcycling option is:

$$\mathbb{E}\Pi^{U\dagger}(x,q) = p_f \cdot \mathbb{E}\min\{D_f^{U\dagger}(S^{U\dagger}(x,q,Y)), q + q_\delta^{U\dagger}(x,q)\} - c_m x - cq - (c+\delta)\mathbb{E}q_\delta^{U\dagger}(x,q,Y) + w\mathbb{E}S^{U\dagger}(x,q,Y),$$

where $S^{U\dagger}(x,q,y)$ and $q_\delta^{U\dagger}(x,q)$ are as described in section EC.1. For any given w, if the firm does not engage in quick response, then it would choose $x = q = x^B$ and obtain the benchmark profit $\mathbb{E}\Pi^{U\dagger}(x^B, q^B) = \mathbb{E}\Pi^B(x^B, q^B)$, which is invariant in β and w. Note that $\mathbb{E}\Pi^{U\dagger}(x,q)$ is continuous and differentiable in x and q. Further, by inspecting the expected profit functions (see EC.2), it is easy to see that for any given x, q, $\mathbb{E}\Pi^{U\dagger}(x,q)$ increases in w and decreases in δ. Therefore, by the envelope theorem, $\mathbb{E}\Pi^{U\dagger}(x^{U\dagger}, q^{U\dagger})$ increases in w and decreases in δ. Since the firm engages in quick response if and only if $\mathbb{E}\Pi^{U\dagger}(x^{U\dagger}, q^{U\dagger}) > \mathbb{E}\Pi^B(x^B, q^B)$, it follows that there exists a threshold $\bar{\delta}^{U\dagger}(w)$ such that this happens if and only if $\delta < \bar{\delta}^{U\dagger}(w)$, and the threshold $\bar{\delta}^{U\dagger}(w)$ is increasing in w. ■

Proof of Proposition 9: The analyses in EC.2 show that there are five solution regions. We prove the result in every region. Specifically, in each region, we consider each feasible solution (x_i, q_i) and show that the result applies, i.e., $x_i \geq x^Q$ and $q_i \leq q^Q$.

In Region A (i.e., $w \geq \alpha p_f$), the potentially feasible solutions (aside from (x^B, q^B)), are (x_1, q_1) and (x_2, q_2). First suppose that (x_1, q_1) is the optimal solution. This solution is obtained by solving $\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial q} = 0$ and $\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial x} = 0$, and is feasible only if $\frac{\partial^2 \mathbb{E}\Pi_1(x,q_1)}{\partial x^2} < 0$. We compare (x_1, q_1) with (x^Q, q^Q) in two possible cases.

First, when $\delta < \bar{\delta}^Q$, recall that $\frac{x^Q}{1-p_f} = \frac{p_f - c - \delta - c_m}{p_f - c - \delta}$ and $\frac{q^Q}{1-p_f} = \frac{\delta}{c+\delta}$. We can rearrange the terms in $\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial x}$ to get

$$\begin{aligned}
\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial x} &= -(p_f - c - \delta)(\frac{x}{1-p_f}) + p_f - c - \delta - c_m + \frac{(w - \alpha(p_f - c - \delta))}{1-\alpha}\left(\frac{x}{1-p_f}\right) + \alpha p_f \frac{(x-q)\alpha(1-\alpha)}{\alpha p_f - p_s} \\
&\quad - \frac{\alpha(w + c + \delta - \alpha p_f)}{1-\alpha}\frac{(\alpha x + (1-\alpha)q)}{1-p_f} \\
&> -(p_f - c - \delta)(\frac{x}{1-p_f}) + p_f - c - \delta - c_m.
\end{aligned}$$

The inequality holds because $\frac{(w-\alpha(p_f-c-\delta))}{1-\alpha}\left(\frac{x}{1-p_f}\right) > \frac{\alpha(w+c+\delta-\alpha p_f)}{1-\alpha}\frac{x}{1-p_f} \geq \frac{\alpha(w+c+\delta-\alpha p_f)}{1-\alpha}\frac{(\alpha x+(1-\alpha)q)}{1-p_f}$. Hence, $\frac{\partial \mathbb{E}\Pi_1(x^Q,q_1)}{\partial x} > 0$, which implies $x_1 > x^Q$.

Second, when $\delta \geq \bar{\delta}^Q$, recall that $\frac{x^Q}{1-p_f} = \frac{q^Q}{1-p_f} = \frac{p_f - c - c_m}{p_f}$. Substituting the equation $\frac{\partial \mathbb{E}\Pi_1}{\partial q} = 0$ into $\frac{\partial \mathbb{E}\Pi_1}{\partial x} = 0$ and re-arranging the terms yields

$$\begin{aligned}
\frac{\partial \mathbb{E}\Pi_1(x,q)}{\partial x} &= -p_f\left(\frac{x}{1-p_f}\right) + p_f - c - c_m - \delta + \frac{w + c + \delta - \alpha p_f}{1-\alpha}\left(\frac{x}{1-p_f}\right) + \frac{\alpha(\alpha p_f - w - c - \delta)}{1-\alpha}\left(\frac{\alpha x + (1-\alpha)q}{1-p_f}\right) \\
&\quad + \alpha p_f \frac{(x-q)\alpha(1-\alpha)}{\alpha p_f - p_s} \\
&= -p_f\left(\frac{x}{1-p_f}\right) + p_f - c - c_m + \frac{w + c + \delta - \alpha p_f}{1-\alpha}\left(\frac{x}{1-p_f}\right) - \frac{w + c + \delta - \alpha p_f}{1-\alpha}\left(\frac{\alpha x + (1-\alpha)q}{1-p_f}\right) \\
&\geq -p_f\left(\frac{x}{1-p_f}\right) + p_f - c - c_m.
\end{aligned}$$

Again, this implies $x_1 > x^Q$. We can use similar methods to obtain $q_1 < q^Q$.

Now suppose the interior solution (x_2, q_2) is the optimal solution. For $\delta \leq \bar{\delta}^Q$, it is easy to see $q_2 < q^Q$, and we have $x_2 > x^Q$ because in this region, $w \geq \alpha p_f > \frac{p_f(\alpha p_f - p_s)}{1-p_s/\alpha - \alpha(1-p_f)} > \frac{(p_f-c-\delta)(\alpha p_f - p_s)}{1-p_s/\alpha-\alpha(1-p_f)}$. For $\delta > \bar{\delta}^Q$, we compare (x_2, q_2) with $x^Q = q^Q = \frac{(1-p_f)(p_f-c-c_m)}{p_f}$. Observe that x_2 decreases in δ while q_2 increases in δ, and thus there exists a threshold $\bar{\delta}_2$ such that this solution is feasible (i.e., $x_2/q_2 > r_3$) if and only if $\delta < \bar{\delta}_2$. It suffices to show that at $\delta = \bar{\delta}_2$, $x_2 > x^Q$ and $q_2 < q^Q$. This is equivalent to showing that as δ increases, when x_2 crosses x_Q from above (or when q_2 crosses q_Q from below), the solution (x_2, q_2) is no longer feasible. To do so, we note that as δ increases, x_2 crosses x^Q from above at $\delta = \frac{(p_f-c-c_m)\left(-\alpha p_f(\alpha p_f - p_s) + (\alpha - p_s - (1-p_f)\alpha^2)(c+w)\right)}{-\alpha p_f(\alpha p_f - p_s) + (\alpha - p_s - (1-p_f)\alpha^2)(c+c_m)} \doteq \bar{\delta}_x$, q_2 crosses q^Q from below at $\delta = \frac{(p_f-c-c_m)(1-p_f)(1-\alpha)(c+w)}{(1-p_f)(1-\alpha)(c+c_m) - p_f(\alpha p_f - p_s)} \doteq \bar{\delta}_q$, and $\bar{\delta}_q > \bar{\delta}_x$. Thus it suffices to show that $x_2/q_2 < r_3$ at $\delta = \min\{\bar{\delta}_x, \bar{\delta}_y\} = \bar{\delta}_x$. Since $x_2 = x^Q$ at this point, this is equivalent to showing $x^Q < r_3 \cdot q_2$, which simplifies into $\bar{\delta}_x > \frac{(w+c)(1-p_f)(p_f-c-c_m)}{p_f(1-p_s/\alpha) - (1-p_f)(p_f-c-c_m)}$. This inequality holds if and only if $\left(\frac{(\alpha-p_s)^2}{\alpha} - (\alpha - p_s)(1-p_f)\right)(w+c) + (1-\alpha)p_f^2(\alpha p_f - p_s) - \alpha(1-p_f)(\alpha p_f - p_s)(c + c_m) > 0$, which holds because in this region, $w + c \geq \alpha p_f$ and $p_f > c + c_m$.

The proof for Regions B, C, D, and E follows similar procedures and are hence omitted. ■